Learn and play with Maths

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Abstract

Mathematics on-line can be an adventure, unusual aesthetic experience, an unexpected enjoyment and a rear pleasure. The presented paper is aimed to address few ways how to discover some of the numerous hidden beauties of mathematical structures. Several historical remarks and new ideas about nowadays trends in mathematical and scientific e-learning are presented. Comments on introducing virtual solutions for mathematics and science education, and about importance of geometric representations and graphical illustrations of mathematical concepts in developed e-learning materials, mathematical e-content and on-line presentations of mathematical scientific papers are given.

Introduction

The third wave of the human civilisation development by creating a new post-industrial society, so called non-visible civilisation of the 21st century. This information and post-information society is characteristic by RECEIVING – PROCESSING – DISSEMINATING of huge amounts of information pieces in the virtual non-visible world by means of rapidly developing new information and communication technologies using new digital media with a limited lifespan and validity. This is the development with many contradictions and ambiguities, as globalisation versus individualisation, data manipulation versus right for true free information , „copy and paste" strategy versus fight for authorship copyrights on original ideas, interest in ready results not in the process of solving problems, gathering pieces of information not complex knowledge, simplification and vulgarisation of the educational process, information boom versus time limits. Daily myriads of bits are appearing on the web, whereas people seek for brief and compound data and graphical representation of data structures. New visual sciences were born and society faces visualisation boom with icons, graphical symbols and logos, flourishing computer graphics and virtual reality, and renaissance of geometry. What impact has this development on science and education? Recently, mathematics is one of those disciplines that fully enjoy and utilise the extreme power of computers not only in connection to their multiple calculation abilities, powerful searching mechanisms, data-mining and e-contents processing, but also as a magnificent tool for scientific modelling, visualisation and manipulations with virtual models enabling their optimisation and continuous re-modelling.

All these changes are reflected also in education and teaching of mathematics, which have recorded a great movement towards utilisation of new information and communication technologies supporting heuristic teaching strategies, investigative approach to obtaining knowledge, development of problem-solving skills and students' self-involvement in the process of learning and education. Interactive activities and on-screen tools allow to visualise mathematical ideas to aid understanding of key concepts and consolidate student's learning. The visualisation of mathematical concepts is highly valuable especially for better understanding and formulae explanations. Most of the existing e-learning activities and solutions enable variegated approach in several levels allowing thus differentiation and better progression for individual students. Topics are revisited throughout certain schemes enabling students to develop understanding of each topic at their individual pace and to the desired deepness and width.

Since the Internet has become a widely used educational environment, education communities have at first adopted a concept of creating educational materials in the form of the complex full-text electronic courses, comprising all course material in large educational modules. Traditionally, content was arranged as a course focused on one particular mathematical discipline at a certain level of difficulty and scope. Currently a new concept for e-learning has been adopted, and it is the object-based learning. This approach is a new way of thinking about instructional content. The idea is to decompose existing course material into smaller units, so-called "learning objects" (1). Learning objects are self-contained, modular pieces of course material appropriately annotated with metadata. They may be combined to form larger educational interactions, e.g. traditional modules, but in a better structured and modular way. The goal is to develop an open architecture for online learning that will allow teaching to be centred on the needs and interests of the learner, enabling learning to occur anytime, anyplace and to allow greater customization and flexibility of the learning environment.

Learning objects have emerged as instructional technology's new paradigm. This idea has gained such broad acceptance that the IEEE has formed the Learning Technology Standards Committee to pursue the creation of common standards for the description, interchange, and management of learning objects (2). Nowadays, the object-based technologies are replacing classical instructional design approaches. This leads to reuse of the instructional materials, and reuse leads to faster development and higher quality instructional contents. In addition, as object-based systems are easier to adopt and easier to scale, different re-assembling of reusable learning objects can create large and very colourful instructional materials. This consequently leads to individual approach and direct addressing of the learner’s needs, and

This paper aims to describe how reusable learning objects might be used in teaching Mathematics with e-technology, in order to enhance better conceptual understanding, and to introduce some from the numerous advantages of the ICT utilisation in teaching. Few comments on the design of data-driven reusable learning object that meets the learners and instructional design requirements is given, followed by several illustrative examples of RLO for modelling in Mathematics, and Geometry particularly.

Mathematics is a highly formalised language of science, which offers many different ways how to represent new phenomena: symbolical modelling, geometric/graphical interpretation of ideas, and illustrative presentation of information, facts and data. Any mathematical co0ncept can be represented in two modes, analytically and synthetically. ICT and e-environment enable combination of both methods, direct access to both representations through animated illustrations and on-line calculations enabling thus brief and compound explanation at hand. Simple example can be seen in fig. 1, which shows a synthetic representation of 3D object, a surface patch. Analytic representation appears in the form of a vector function in two variables defined on the unit square

$$\mathbf{r}(u,v)=\begin{pmatrix} x(u,v)=-(5+5\cos 2\pi u)\sin 6\pi v, \\ y(u,v)=(5+5\cos 2\pi u)\cos 6\pi v\cos 4\pi v-5\sin 2\pi u\sin 4\pi v+5(\cos 4\pi v-1) \\ z(u,v)=(5+5\cos 2\pi u)\cos 6\pi v\sin 4\pi v+5\sin 2\pi v\cos 4\pi v+5\sin 4\pi v \end{pmatrix}, \quad (u,v)\in\langle 0,1\rangle^2$$

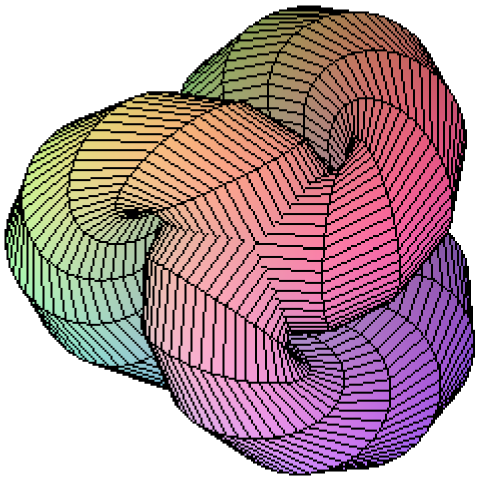

Fig. 1 View of a surface in 3D

A continuous movement in the space, synthetically represented as a geometric transformation of revolution about line can be analytically represented by a square matrix function that is regular for any value of variable *u* from the unit interval

$$\begin{pmatrix} \cos 2\pi u & \sin 2\pi u & 0 & 0 \\ -\sin 2\pi u & \cos 2\pi u & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{for } u\in\langle 0,1\rangle$$

or as quaternion

$$\begin{bmatrix} \cos 2\pi u & \sin 2\pi u \\ -\sin 2\pi u & \cos 2\pi u \end{bmatrix}.$$

Traditional Mathematics and Geometry courses at European technical universities consist from the following topics:

- Basics of Linear Algebra (systems of algebraic equations, matrices, determinants and vector spaces)
- Calculus + Multivariable Calculus (differentiation and behaviour of functions of one and more variables, optimisation problems, calculations of indefinite and definite integrals and applications, multiple integrals)
- Basics of Differential Equations (easy analytic solutions of ODE)
- Numerical mathematics (iterations, interpolations and approximations, numerical methods for solving ODE and PDF)
- Basics of projection methods and 3D geometry (Monge method, axonometry, linear perspective, stereographic mapping, elementary geometric figures)
- Differential geometry of curves and surfaces used in technical practice
- Geometric modelling of free-form figures (curves, surfaces, solids) in CAD systems

Additional topics incorporated to mathematical courses due to new trends and needs:

- Mathematical and geometric modelling using fuzzy approach and stochastic methods
- Numerical interpolation and approximation methods for calculating solutions of problems virtually representing and modelling real-world situations
- Techniques for visualisation of abstract virtual models and data structures
- Concepts of geometric spaces (also non-Euclidean), geometric transformations (including non-linear deformations) in virtual space and manipulations with objects

Internet and on-line solutions, computer graphics and virtual reality modelling helped to establish our age to be the age of visualization and animation, in which people facing the graphical information boom worship the culture of pattern recognition. Logos, icons, graphical marks and indicators appear everywhere as a common communication tool. Usually the first and often also the last received information on an arbitrary phenomenon is visual. Virtual reality experience, virtual modelling, interactive manipulations with virtual models enabling continuous movement, deformations and shape optimisation open the doors to more complex and comprehensive educational and acquisition processes.

TODAY MORE GEOMETRY IS NEEDED THAN EVER BEFORE!

Geometry is a specific European

- dimension
- attitude
- way of understanding
- recognition and differentiation key
- distribution pattern
- characteristics
- art and science

that is typical for European civilisation and deserves to be preserved and further cultivated.

In the historical development of mathematics as a science, geometry played a crucial role from the very beginning. In the ancient Greece (from about 600 B.C.) it was cultivated by work of many scientific personalities as Thales, Pythagoras, Eucleides (who developed the very first axiomatic system of geometry as science in his book Stockeia – Elementa, 300 B.C.). Later Plato came with the concept of regular (platonic) solids related to philosophy, Archimedes calculated the area of a figure, which was surrounded by parobola (curve) and chord (straight line) and he gave the backgrounds of calculus with his exhaustion method. Apollonius studied conic sections already in 200 B.C., while Hypathia from Alexandria (fig. 1), 370 – 415, edited the work *On the Conics of Apollonius*, which divided cones into different parts by a plane. This concept developed the ideas of hyperbolas, parabolas, and ellipses. Hypatia with her graphical work on this important book, made the concepts easier to understand, thus enabling the work to survive through many centuries. Hypatia was the first woman to have such a profound impact on the survival of early thoughts in mathematics. These deep geometric results were reflected in the flourishing Greek art and architecture.

Fig 1. Hypathia from Alexandria, 370 – 415.

Latex on in renaissance in Italy, fine artists were not only skilled and talented outrages painters, but also scientists and architects, engineers and designers, technicians and educators.

Leonardo da Vinci, (1452-1519), an all-round genius whose paintings and inventions changed the world, addressed in his work many theoretical geometric problems of perspective, problems with light and shadow in optics, topics about human body related to biology and medicine. He created beautiful architectural structures and invented various mechanisms and machines technically perfect and ingenious, and with a real touch of industrial designer eyes. Renaissance in Germany represented by the personality of Albrecht Dürer (1471-1528), German artist, scientist - mathematician, developed the scientific backgrounds of basic linear perspective and geometry of projection methods in general. In his work, he found the idea used also nowadays in computer graphics, leading to free-form modelling of curves and surfaces by means of interpolation and approximation methods, see fig. 2. Dürer´s masterpiece Melencolia I is the most fascinating allegory of Geometry bringing together different issues and unsolved problems of that time, such as correct perspective drawing, platonic solid dodecahedra and its perspective view, perspective views of a ball and mill stone in the from of a cylinder, depicted skew ladder with ? in line segments joining points of two skew lines, and also the famous magic square and special drawing instruments (compasses and ruler).

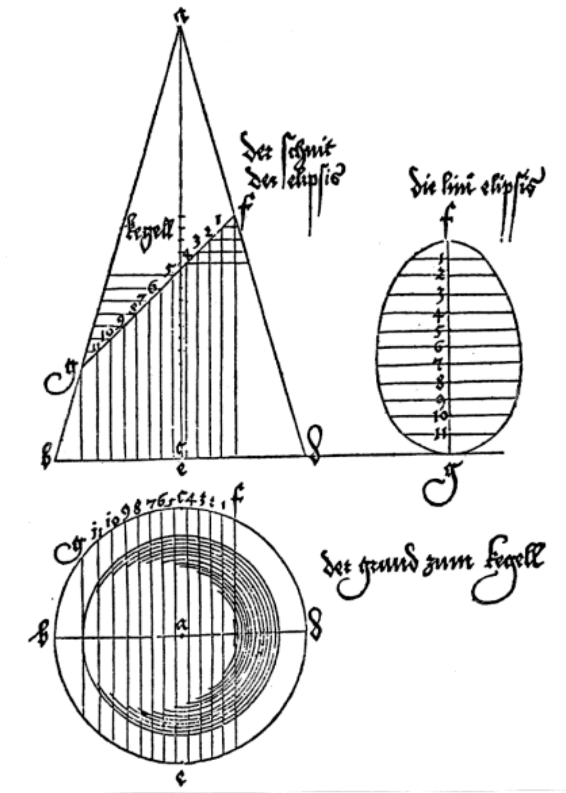

Fig. 2. Albrecht Dürer: Letters III

Light and shadow in Rembrandt and Velasquez paintings are tools used to create the illusion of 3D space, while their masterpieces are phenomenal examples of virtual 3 dimensional scenes elaborated by hands, but not less perfect as contemporary virtual scenes made by means of computer graphics.

Non-Euclidean geometry is apparently present in modern art masterpieces of Cézanne, Braque, in the cubistic school of Picasso, and geometry of transformations plays a vital role in the work of Dali or Gaudi. Chaos and disorder appear in the modern art of Klee, Delaunay or Kandinski (fig. 3). M. C. Escher pieces of work are directly representing mathematical concepts of knots, hyperbolic mosaics, dense layout and space filling of hyperbolic plane, and also non-existing objects represented as views of objects from the four dimensional space.

Fig 3. Vasili Kandinski: Composition VIII (1923)

Geometry plays important role in all sciences, physics particularly. Einstein theory of relativity is based on idea of the non-Euclidean space, namely the Riemann-Minkowski geometry of the four dimensional time-space. Symmetry and regular polyhedra are basic terms in chemistry, crystallography. Living objects of study in biology poses their form, which is basically a geometric figure. Nowadays geometry can find numerous direct applications in many new sciences, as robotics and mechatronics, scientific visualisations, superstring theory and M-theory. Global positioning systems GPS and digital mapping and drawing with GPS tracks are based on pure geometric algorithms.

Future development is also opened in connection to computer graphics and CAG design and modelling, Virtual reality, where animated geometry and interactive manipulations and on-line modelling are taken for granted serve as examples of a vital presence of geometry and geometric relations in any environment.

There are many different solutions for interactive Maths on-line presentation, e.g.

- doc, pdf and html files frequently used for on-line solutions in the previous years are today very rapidly substituted by xml files with embedded MathML coding that introduces to the web live maths formulas with their semantics
- Flash animations
- on-line Java animations (produced in Cabri or Geogebra)
- webmathematica java server pages
- Maple maplets
- Virtual Lab Toolkits (virtuallab.tu-freiberg.de, Scientific talk, Whiteboard)
- On-line drawing tools (www.plot3d.net, kmatplot.sourceforge.net)
- Virtual reality VRML files

Many different solutions exist also for the organisation of Maths on-line. Some of them are in the following forms:

- Electronic books
- E-learning modules (dmath.hibu.no/xmath/pilotcourse)

- Information portals with hyperlinks (www.evlm.stuba.sk)
- Educational e-learning portals with maths support resources (www.mathcentre.ac.uk)
- Databases of e-learning materials (www.evlm.stuba.sk/databasemenu/menu_files/frame.htm)
- Links to free web resources (demonstrations.wolfram.com)

Many programme schemes exist at the European level supporting development of different solutions for e-learning in Maths and interactive on-line educational resources

- Socrates, Minerva, Erasmus Mundus programmes
- Tempus IV – Higher Education Cooperation (2007 - 2013)
- Ceepus II - university study programmes for Central and Eastern Europe (2005 - 2011)
- Lifelong Learning programmes
- Education and Culture (former Leonardo da Vinci)
- eContent+
- UNESCO – Education for All by 2015

Aims of the projects related to presentation of mathematics on-line are usually the following:

- Tu use uniform format of presented variety of e-learning materials
- To adopt semantic xml files + MathML coding of mathematical formulas
- To include animations, on-line manipulations and modelling
- To illustrate learning material with applications
- To provide hyperlinks to the supplementary materials on web
- To solve problems using the Step-by-step mode of solutions

- To collect sets of problems in particular maths topics with provided results
- To introduce interactive on-line calculations and live graphics – jsp files
- Tests – electronic forms
- To develop interactive textbooks of mathematics

One of the successful projects, which has been coordinated in Slovakia by the Slovak University of Technology in Bratislava, was the Leonardo da Vinci project EVLM – European Virtual Laboratory of Mathematics for years 2006 – 2008. Project team consisted from 9 partner universities in 8 European countries: Slovakia, Bulgaria, Czech Republic, Finland, Hungary, Ireland, Spain and United Kingdom. Central portal of EVLM in English provides links to 7 national portals presenting all information in national languages of partners. All portals host Databases of available e-learning source materials from Mathematics in relevant languages, consultation centres with free present or on-line consultation services, and two guide books providing advice on how to use and develop e-learning materials by means of mathematical software products and solutions.

Presented materials developed in the frame of the project European Virtual Laboratory of Mathematics are freely accessible on web (www.evlm.stuba.sk). The aim of the project was an optimal usage and synergy of the pedagogical methods applied in a classical educational process with teachers in the classroom, and the brand new information and communication technologies including internet in a new form of a virtual classroom.

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